Journal of Arts & Humanities

Volume 08, Issue 01, 2018: 68-75
Article Received: 19-01-2019
Accepted: 29-01-2019
Available Online: 31-01-2019
ISSN: 2167-9045 (Print), 2167-9053 (Online)
DOI: http://dx.doi.org/10.18533/journal.v8i1.1577

Exploring the Complementarity of Mathematical and Artistic Thinking in Design through 3-D Printing

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ABSTRACT

Most development of the “STEAM” (science, technology, engineering, arts, and mathematics) paradigm has been in the realm of education; however, the idea of combining technical and artistic thinking has many practical applications in product design. The more artistic side of technology design is often considered during design, but it tends to be subservient to the technical (“quantitative”) side due to the lack of a definite method for combining the technical and artistic concepts. New developments in 3-D printing and related technologies have opened some doors toward this by allowing the qualitative and quantitative design aspects to be expressed via a “common language” derived from the complementarity of mathematics and art. In this article, the complementarity of thought in design is discussed through the medium of 3-D printing. The basic concept of a “complementary design thinking” approach and work-flow of 3-D printing is explored in detail, with a focus on the value of the technology to bring together both the quantitative and qualitative approaches to design thinking. A case study using a 3-D printed model of the well-known Costa Minimal Surface from mathematics is presented and discussed to demonstrate the concepts. The concepts presented have clear policy implications for design, arts, education, engineering, and human-technology interaction research and practice; these implications are discussed in detail.

Keywords: Art and Design, Mathematical Art, Steam, Technology in the Arts.
JEL Classifications: C02, C61, L60, O31, O35.

1. Introduction

The modern focus on high-technology and rapid-turn-around product design has served to advance the standard of living throughout the world in many ways; however, the over-reliance on metrics has also contributed to a decline in the appreciation of quality, beauty, and intrinsic value in the lives that

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many of these technologies support (Younes & Al-Zoubi, 2015; Gultekin 2004; Ramey 2014). Personal technology items, such as computers and phones, are made to look aesthetically pleasing on the outside and work very well – but only for a relatively brief life. They are generally consumable, rapid-obsolescence items which are meant to be "burned up" and discarded quickly so the consumer can move on to the next model or upgrade (Hodges & Taylor, 2005; Park 2010). Both the rise of extreme metric-based ("high-tech") design and the increasingly consumerist culture contributed to the disappearance of high-quality products and it seems to be causing something of a devolution of culture and loss of the innate human sense of value, quality, and beauty (Hyland, 2013; Goodwin et al., 2008; Bownes, 2017).

It cannot be denied that this focus on technology and mathematical thinking in design has advanced technology at a rapid pace, but human interaction must also be taken into account in terms of quality, ease of use, culture, and physical appearance. The idea of purely artistic or qualitative design has generally become reviled by many people as "useless" or "not profitable" (or even worse, "political"), even though it was the source of most of the culture and technology that the modern world enjoys (Wilson, 2002). The holy grail of design is an effective method for taking advantage of the complementarity of the two approaches, considering them both and taking value from each to produce the best product, both from a technical and user-centric perspective.

The concept of "design thinking" (Henriksen, 2017; Dorst, 2011) is an interesting development within both technical and artistic design that has come more into prominence in recent years. It can be said that one purpose of "design thinking" is to escape the reliance on "quantitative" (Sen, 2004) versus "qualitative" (Saldana, 2015) design approaches and combine the best of both (Henriksen, 2017) in some form. While this has been done effectively in some cases with heuristics-based design for practical design problems (Chong et al., 2009), there is still not an effective formal method for considering this. Each design thinker has a different approach to solving design problems, so there is no standard set of guidelines to generate a design; consequently, there are many ways to come up with the same product with the same technology that executes the same function but each with a different design. The recent rise of 3-D printing and other "additive technologies" in recent years may open a window for exploring this, however. These technologies, based on 3-D computer modeling and direct manufacturing products, offer a link between the two and allow a more effective exchange of concepts between them. Even if 3-D printing is not used to manufacture the final product, its conceptual framework is a useful tool to explore and refine the idea of design thinking. The purpose of this article is to examine this link and explore the conceptual and practical approach for taking advantage of it to improve design thinking.

The methodology for this work was simple: First, the need for complementarity in design thinking was established via a literature search and experience of the authors. Next, the design impact of 3-D printing technologies was analyzed, showing that a clear link (the digital model) exists between the work-flow of 3-D printing and the practical use of complementary design thinking in several domains. Finally, a case study was developed to show this link and demonstrate the approach taken to practically implement the complementary design thinking. It was found that the use of 3-D printing indeed provided a useful and simple tool for practically utilizing these concepts, one that is not dependent on any particular domain or starting design approach. This work has several important policy implications for design, arts, education, engineering, and human-technology interaction research and practice, which will be discussed in detail later in the paper.

This article examines the concept of complementary design thinking (Section 2), the use of 3-D printing technologies to advance this approach (Section 3), a case study using the Costa Minimal Surface to show the mechanics of the complementary approach (Section 4), research and practice policy implications of the complementary design approach (Section 5), and finally, conclusions and suggested future work in this area (Section 6).

2. Understanding complementary design thinking

Under the concept of design thinking presented in this work, it is assumed that the quantitative ("technical") tasks of design and the qualitative ("artistic") side would be completed by separate teams of designers with different training and educational backgrounds. While there are some products (such as art pieces) that would be designed completely by qualitative designers and some (like computer processors or auto parts) that would be done completely by quantitative designers, it is typically assumed that some aspects of both would go into the design of most consumer products. Finding a way to link these two worlds together in a complementary way is the goal of *complementary design thinking* as presented in this article. The adoption of this thinking approach would be very helpful to the push for a "STEAM" (science, technology, engineering, arts, and mathematics) paradigm, as it would provide a framework for more rigorous and wide-spread adoption of artistic thinking into the traditional "STEM" world.

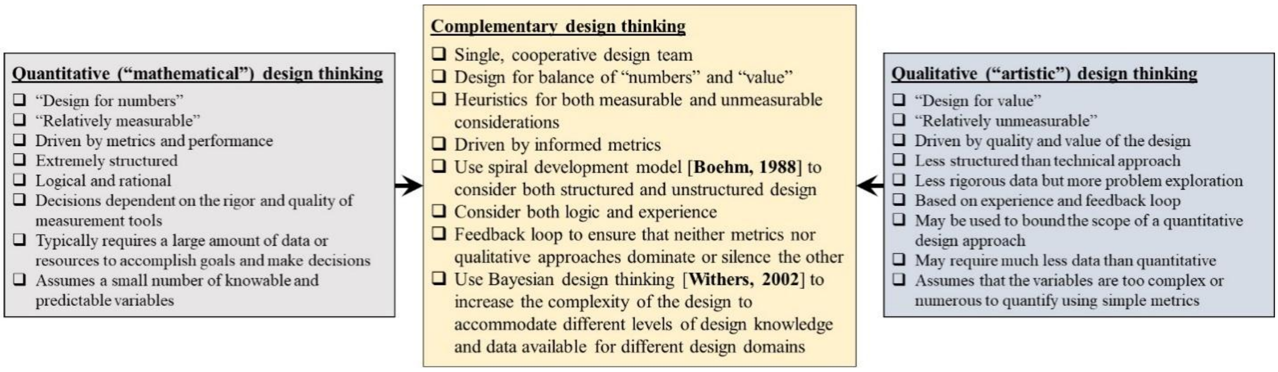

Figure 1: Some aspects of quantitative, qualitative, and complementary design thinking approaches and their relationship

Figure 1 shows some of the basic concepts behind each domain and how they might be combined into a single concept. The defining goal of this proposed approach to complementary design thinking is to gain inputs into design decisions from both the "mathematical" and "artistic" sides in order to leverage both to produce the best possible design. Toward this end, efforts should be made to integrate the design perspectives together whenever possible. This may be aided greatly by structuring the design process by:

- ***Goal 1****: Attempting to balance the reliance on metrics with more qualitative criteria. This may take the form of "bounded" metrics, where the interpretation of the metrics is guided by qualitative criteria, or the form of a multi-objective decision problem with some of the decision variables are in the quantitative and some in the qualitative realms.*
- ***Goal 2****: Using a mixture of heuristics and mathematical models in the product design, in order to capture both the measurable (i.e. speed, mass, life-to-failure, etc.) with unmeasurable (i.e. beauty, quality, cultural influence) of the design aspects*
- ***Goal 3****: Using a spiral development model (Boehm, 1988) or similar tool in product design in order to be able to better control risks and uncertainties associated with using heuristic or non-metric bases design criteria*
- ***Goal 4****: Considering both the logic of the design process and the experience of the designers and stakeholders as equally valuable and worthy of driving design decisions*
- ***Goal 5****: Using a good feedback loop and common goals and "common language" within the design process to ensure that none of the perspectives are suppressed or considered to be less important than others*
- ***Goal 6****: Using Bayesian design thinking (i.e. update the assumptions and goals as more information becomes available to the designers) (Withers, 2002) to move the design forward toward the final product*

This is hardly an exhaustive list of the ways that the design process can use complementary design thinking, but it is a good start. Ways to practically implement these into a general design method are still elusive, but may be helped greatly by the advent of 3-D printing and other technologies which bridge the gap between science and art.

3. Complementary design thinking and 3-D printing

A main reason that 3-D printing and its related technologies has received so much attention in recent years is that it bridges the gap between science and art. Since it fundamentally gives one the ability to create almost anything directly from a computer model, it is something that both the technical and artistic worlds can appreciate and use to their advantage. Many different variations of the technology exist, but this article will focus on the general case, considering the common fundamentals and not the specific manufacturing processes. Figure 2 (Chadha et al., 2018) below shows the basic process path for all 3-D printing processes, from computer model to finished product.

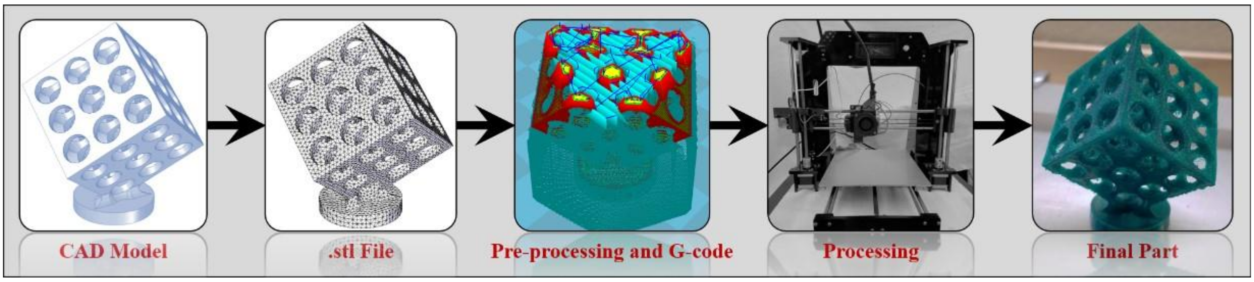

Figure 2: Basic 3-D printing process map [Chadha et al., 2018]

The greatest value offered by 3-D printing technology is its position between technology and arts, a unique position that offers much promise for its use in facilitating complementary design approaches. The link between the two is the computer-aided design (CAD) model, which can be produced using either a mathematical or artistic approach or some combination of both (Segerman, 2016). As shown in Figure 2, the completed CAD model is the starting point and driver for the entire 3-D printing process; the other steps are relatively automated and driven directly from the CAD model. Therefore, the model is the "link" between the various design methods and the production process within 3-D printing technologies, as shown in Figure 3.

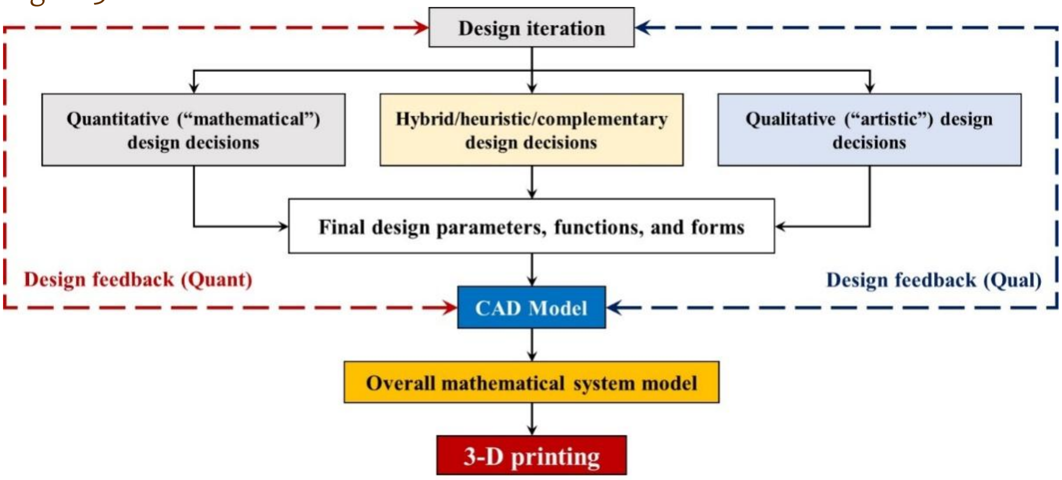

Figure 3: Link between design perspectives, CAD model, and 3-D printing

This model may be created using by scanning objects, via a freeform sculpting tool, generated using a mathematical function, or a variety of other means. Its most important aspect in terms of design thinking is that that the model will be "directly-describable" from either the technical or artistic perspective, providing a tool for interpreting the design concepts from either view. This common modeling method, coupled with the ability to mesh (approximate any object in terms of a mathematical model) and directly manufacture the object, provide a good, practical framework for complementary design thinking to be used in practical design. A good way of thinking about this is that *the additive technology gives the ability to view a product or object as either a sculpture or mathematical model and which could be understood as either or both*, depending on the need. Unlike most technologies, 3-D printing has the added benefit of being highly automated and requiring little specific technical knowledge or experience from the user, making it available for any designer or design team to take advantage of.

4. Case study: 3-D printed costa's minimal surface

One of the interesting recent applications of 3-D printing has been in the visualization of mathematical models, both for education and art (Segerman, 2016; Melko, 2010). These visualizations of mathematical models, particularly parametric models, make interesting case studies in complementary design thinking, as they can be driven by or applied to both technical design and arts. A very interesting type of parametric model is the minimal surface, a type of surface that locally minimizes its area and is in effect an "optimal" surface under its surrounding conditions. Examples of minimal surfaces include egg shells, honeycombs, sea shell, bubbles, leaves, various biological structures, crystals, membranes, and many other common things (Andersson et al., 1988; Taylor, 1976). These natural structures commonly drive inspiration in art and design, while also being describable mathematically, so minimal surfaces are a very useful and intersectional topic of study for complementary design thinking.

In simple terms, a parametric function is one that describes a function in terms of various adjustable parameters or “knobs” within the equations (Stover & Weisstein, 2018). The parametric function selected for this case study is the famous Costa’s Minimal Surface, which can be described by (Weisstein, 2018):

$$x = \frac{1}{2}R\left\{-\zeta(u+iv) + \pi u + \frac{\pi^2}{4e_1} + \frac{\pi}{2e_1}\left[\zeta\left(u+iv-\frac{1}{2}\right) - \zeta\left(u+iv-\frac{1}{2}i\right)\right]\right\} \quad (1)$$

$$y = \frac{1}{2}R\left\{-i\zeta(u+iv) + \pi u + \frac{\pi^2}{4e_1} - \frac{\pi}{2e_1}\left[i\zeta\left(u+iv-\frac{1}{2}\right) - i\zeta\left(u+iv-\frac{1}{2}i\right)\right]\right\} \quad (2)$$

$$z = \frac{1}{4}\sqrt{2\pi}\, ln\left|\frac{\phi(u+iv)-e_1}{\phi(u+iv)+e_1}\right| \quad (3)$$

$$\zeta(z) = Weierstrass\ zeta\ function \quad (4)$$

$$\phi(g_2, g_3; z) = Weiserstrass\ elliptic\ function \quad (5)$$

The variables $x, y,$ and z describe the location of the surface walls in a 3-dimensional space (Figure 4a) (Melko, 2010), while the other variables and functions are the “knobs” which can be adjusted to make the minimal surface conform to the needs of the user, whether for some kind of architectural structure (Velimirovic et al., 2008) or an art piece (Nylander, 2018). Using typical variables, this surface can be plotted, converted into geometry, and 3-D printed (Melko, 2010) relatively easily, as can be seen in Figures 4a and 4b (Fields, 2018). One aspect of minimal surfaces is their ability to be defined in terms of genus, or the “number of holes” (Traizet, 2008) (the standard Costa surface has genus 1 with three punctures (Costa, 1984; Weisstein, 2018) but can be defined as a higher genus (Nylander, 2018), further allowing them to be modified and adjusted as needed by the user.

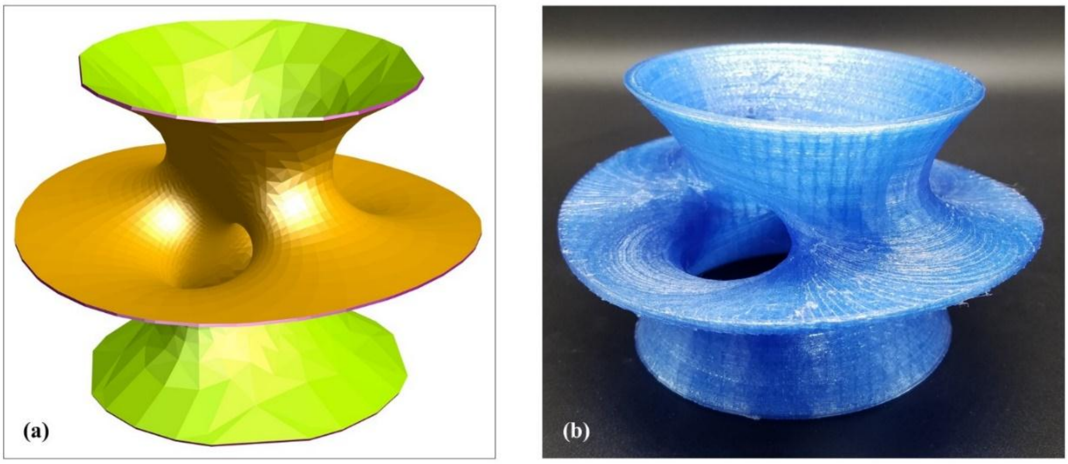

Figure 4: (a) Costa’s Minimal Surface plot example (Melko 2010) (used with permission) and (b) example modeled and meshed by (Fields 2018) and printed by the authors (FDM, 0.20mm resolution, Makergeeks crystal blue PLA with break-away support)

The mathematical minimal surface is certainly a good way to study the intersection between mathematics and art, as the goals of complementary design thinking can be clearly and easily realized with this kind of problem. Looking at each of the goals (excluding Goal 3, as it does not apply to this case study) for the Costa minimal surface:

- ***Goal 1: Balancing metrics and qualitative criteria.*** *A product based on minimal surface geometry can be designed using both metrics and qualitative criteria (i.e. “heuristics”). The set of parametric equations retain their form, but the parameters can be adjusted. The “quantitative” aspects of such a design would be driven by the choice of minimal surface and the “qualitative” aspects by the choice of parameters. Both aspects would determine the acceptance limits on the ranges of the allowed parameters.*

- ***Goal 2: Using a mixture of mathematical models and heuristics****. A spiral development model for such a product would be appropriate, as the choice of which minimal surface to use (of the many in existence) and the subjective values of the parameters may need several iterations (based on Bayesian thinking, each should be driven by new or improved design information) to obtain an "optimal" design. However, each of these iterations or intermediate designs are valid designs in their own right, hence the use of the spiral model instead of just a standard iterative model (Zeigler & Muzy, 2016) which gives only one valid design at the end of the process.*
- ***Goal 4: Consider both logic and experience.*** *The logic of the design process would be driven by the mathematical model or models behind the minimal surfaces, while the experience of the stakeholders would drive the choice of parameters and possibly of the class/genus of minimal surface used in the design.*
- ***Goals 5-6: Use a feedback loop and update as new information is gathered.*** *As previously discussed, the use of minimal surfaces crosses boundaries and overlaps into both mathematics and art, providing a basis for a "common design language" and effective feedback loop during design*

5. Research and practice policy implications

The adoption and wide use of complementary design thinking could have significant policy implications in several domains, including design practice, design and mathematics education, arts practice and education, engineering, and human-technology interaction. While not an exhaustive list, some of the major potential impacts are:

- ***Design practice****: A better integration method for the quantitative and qualitative aspects of design would result not only in superior designs, but also facilitate communication between designers. This would allow the construction of better-functioning interdisciplinary design teams who could tackle larger and more complex design problems in an orderly way. It would also help to bridge trust gaps between artistic and technical design approaches, helping to convince engineers and technical designers to better value the qualitative side of design and helping to communicate the practical aspects, limitations, and business-related trade-offs of the designs to the artistic-minded designers.*
- ***Design, engineering, and mathematics education:*** *Complementary design thinking is one of the avenues for implementing and growing the STEAM paradigm in education. The complementary approach will facilitate communication between students and teachers, helping to bridge understanding gaps between different kinds of learners (Gardner, 1994) and helping to facilitate a more equitable educational outcome in both the arts and in STEM/STEAM fields.*
- ***Arts practice and education:*** *The impact on practice and education in the arts is similar to that in the design and mathematics education. The integration of mathematical thinking into the arts can only have benefits to the production of art, while also helping to better communicate concepts to students. If desired by the artist, this could provide a method for producing more standardized and replicable art; while some artists could decry this as reducing artistic freedom, it does allow the cheaper and quicker production of pieces, allowing more people to see and enjoy the art. In addition, art pieces, including fully free-form ones, could be "cataloged" and modeled more easily using mathematical techniques, allowing easy production of additional copies should the demand arise, or the original piece be damaged.*
- ***Human-technology interaction:*** *While the use of complementary design thinking would help to integrate the "human" side of users into technology, the greatest benefit comes via the use of the 3-D printing technology. Both professional and amateur artists and designers could use these technologies to produce works for their own use and enjoyment, helping to improve their own design thinking while exploring effective ways to interact with the technology.*

6. Conclusions and future work

Much research and concept refinement are needed to develop a practical way to apply this concept of complementary design thinking. However, directly considering the intersection of art and mathematics is a good place to start, as demonstrated by the case study presented 3-D printing is an excellent tool to facilitate this fusion of ideas, as it provides an intersection that is useful to both sides and serving to drive and be driven by developments in both design perspectives. The several policy impacts of this work point to its usefulness and the need for this type of approach in design thinking, one that is interdisciplinary and not dependent on a particular field, technical background, or design method used.

Future work in this area should focus on the practical implementation of these ideas, both from the technical perspective and from the artistic perspective. From the experience of the authors, the most difficult task will be convincing designers with different perspectives to collaborate and value each other's ideas. It will be difficult to change the standard practice, but the potential benefits are great and certainly make the concept of "complementary design thinking" worth pursuing. Expanding the use and knowledge of 3-D printing technologies should aid in this pursuit, as users should be more willing to change their practice and approach if they can see tangible benefits. This technology will aid in other researchers' pursuit of ways to do this and build upon the ideas discussed in this article.

7. Acknowledgements

The authors thank Mike Melko (Mathematical Synergistics) and George Beck (Editor for *The Mathematica Journal*) for providing access to Figure 4a and for their discussion about this topic. Thanks to the anonymous reviewers for their helpful and constructive feedback on this work. All conclusions and opinions in this paper are solely those of the authors. No external funding was used to complete this work nor fund its publication.

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